

Spinning Rocket Simulator

Turntable Design

A Senior Paper

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-ABSTRACT

Contained herein is the research and data acquired from the Turntable Design portion of the Spinning Rocket Simulator (SRS) project. The SRS Project studies and eliminates the effect of coning on thrust-propelled spacecraft. This design and construction of the turntable adds a structural support for the SRS model and two degrees of freedom. The two degrees of freedom, radial and circumferential, will help develop a simulated thrust force perpendicular to the plane of the spacecraft model while undergoing an unstable coning motion. The Turntable consists of a ten-foot linear track mounted to a sprocket and press-fit to a thrust bearing. A two-inch high column grounded by a Triangular Baseplate supports this bearing and houses the slip rings and pressurized, air-line swivel. The thrust bearing allows the entire system to rotate under the moment applied through the chain-driven sprocket producing a circumferential degree of freedom. The radial degree of freedom is given to the model through the helically threaded linear track. This track allows the Model Support and Counter Balance to simultaneously reposition according to the coning motion of the Model. Two design factors that hinder the linear track are bending and twist due to torsion. A Standard Aluminum "C" channel significantly reduces these two deflections. Safety considerations dictate the design of all the components involved in this project.

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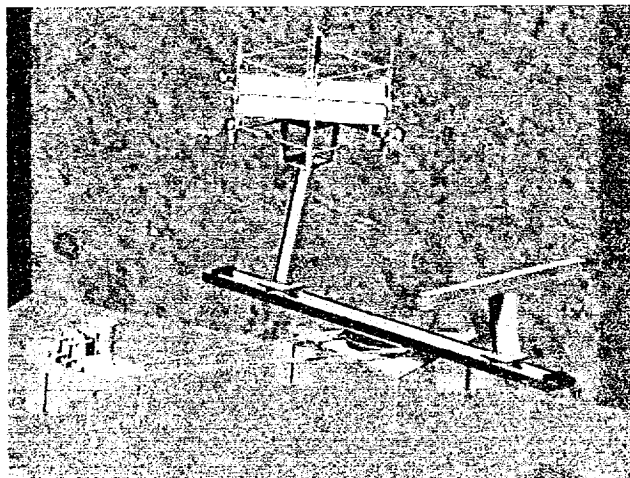


Figure 0 - Solidworks® Rendering

EXECUTIVE SUMMARY

The Spinning Rocket Simulator Project will study and eliminate the effect of coning on thrust-propelled rockets. At the present time, NASA controls the coning of spinning spacecraft using active thrusters. Springs and dampers offer a more economic solution by passively attenuating the unstable coning. NASA sponsors this design of a spinning model in order to obtain a viable solution. Over the past few years, Oral Roberts University students have designed the frictionless air bearing, spacecraft model, RF communications system, control systems, and other essential components (Figure 0). This year's Turntable Design Project has involved the design and construction of the Model Supports, Counter Balance and Turntable. The Turntable consists of the Linear Track, the Driving Sprocket, the Air- Line Swivel, Slip Ring housing and Thrust Bearing. This project adds a structural support for the SRS model and adds two degrees of freedom. These two degrees of freedom, radial and circumferential, help develop a simulated thrust force perpendicular to the cross-sectional plane or

mounting plates of the spacecraft model while it experiences the coning instability.

The design criteria resulted in a variety of analyses and tests in order to adequately support the Spacecraft Model and Counter Balance. The physical properties of the linear track, calculated through angle of deflection equations, dictated the strength considerations of the Linear Track Base Plate. The design of this Base Plate, specifically the placement of the structural bolts, brought about the ensuing design of the Driving Sprocket, the Spacer and the Thrust Bearing Column. The Linear Track Base Plate, designed using an American Standard "C" Channel, deflects only 0.166 in over its entire 126 inch length with a stiffness parameter of 10.5×10^6 psi. The turntable system weighs ??????? and is dynamically balanced about the global axis of rotation. The Thrust Bearing allows the entire system to rotate with little friction at the design speed of 1 Hertz. The entire turntable system now operates under the parameters and will soon be integrated with the rest of the components of the SRS Project.

PROJECT DEVELOPMENT

Throughout the last year, the design and concepts of the Turntable System have evolved due to manufacturing considerations as well as changes in the exact design criteria. The initial design of the Turntable System, as envisioned by Damon Bennett¹ (Figure 1), consisted of a Linear Track, a driving Sprocket, a set of slip rings, and an airline swivel. This design inspired the final production of the Turntable

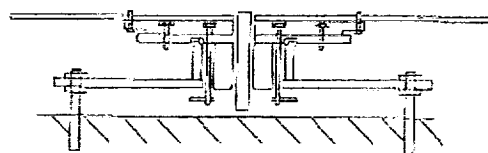


Figure 1 - Envisioned Turntable system

¹ Former ORU Engineer, Mechanical Emphasis

system. The first challenge was to determine the physical properties of the Linear Track, the modulus of elasticity, equivalent area moment of inertia, and deflection. The area moment of inertia was determined to be $3.35 \cdot 10^{-4} \text{ ft}^4$ ($2.89 \cdot 10^{-6} \text{ m}^4$) by assuming the Linear Track to be a rectangular solid. In order to obtain the modulus of elasticity, the Linear Track was suspended in the center by a hydraulic jack. The jack lifted the Track till only the ends touched the table on which it rested. Through knowing the weight of the Track 140 lbs (63.5 kg), and the deflection at the center 0.157 in (4mm), the modulus of elasticity was found to be 1856 ksi (12.8 GPA). The determination of these two properties allowed for the angle of deflection to be calculated for a "worse case" loading. This loading occurs when both the Model Support and Counter Balances extend to the extremities of the Linear Track and produces a force of 124.1 lbs (552 N). The moment produced by this force, 651.5 ft*lb (883.3 m*N), bends the Linear Track 0.5 in. (1.4 cm) elongating it beyond the original design criteria of 0.5° .

The original conception employed a flat steel plate to strengthen the Linear Track; this design, however, became impractical due to manufacturing considerations. In analyzing the Linear Track, the design criteria, less than 0.5° of deflection and 1.0° of twist, dictate that the inertial forces be found for a worse case scenario. This design specification is determined by assuming that the Track is rotating at its maximum angular velocity of 1 Hz and comes to an "immediate" stop. The time from full velocity to zero velocity is determined to be the time for the chain to tighten completely as

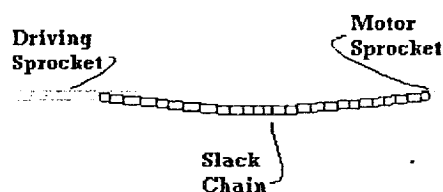


Figure 2 -Sprocket & Chain Slack

though the chain had wrapped around something and had halted. This slack (Figure 2) would tighten on the Driving Sprocket in 6.91m sec. Using this time, the half-length of the track (56.42 in.), the calculated height of the Model Support (31.9 in.), and the measured weight of the entire Model Column and SRS Model, the impulse at the end of the track is determined. The torsional moment applied to the end of the Track is 1.63×10^5 ft*lb (67.4 m*kN) which, being as large as it is, produces a 31.8° angle of twist when no structural support exists (see Appendix A). The flat steel plate was then added and it was found that to limit the angle of twist to 1.0° the steel plate would need to be 3.75 in thick (see Appendix A). The manufacturing and dynamic considerations of this 907 lb steel plate necessitated an innovative design. Through research it was determined that a "C" channel would meet the design criteria and weight limitations. The program Maple® calculated the angle of twist and deflection (see Appendix C) for two different types of steel American Standard C Channels. Though this design substantially decreased the weight and angle of twist it still weighted too much. Though the design criteria did not specify a minimum weight, the less mass rotating at 1 Hz the better. Therefore the final design was an Aluminum Standard "C" Channel that weighed about 50 lbs (see Appendix D). Originally supporting struts were required to decrease the angle of twist, however, because of the new "C" Channel design, the struts were eliminated. The reason for this is that the Linear Track by itself only twists 6.82° and deflects only 0.573 in. (1.46 cm) after applying the large design torque (see Appendix C ,Actual Design).

After determining the material and structural properties, Solid Works® is used to design the bolt patterns that interface between the Linear Track Baseplate (LTBP), Spacer, Driving Sprocket, and integrated other components of the

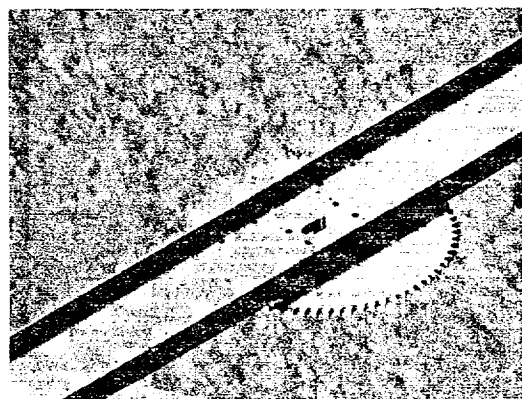


Figure 3 - LTBP, Spacer, Driving Sprocket

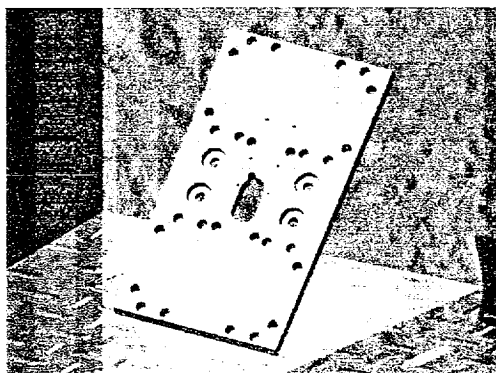


Figure 4 - Spacer

Turntable System (Figure 3). The bolt patterns, after being positioned on the LTBP, are simply copied to the Spacer and Sprocket. The purpose of the Spacer (Figure 4) is to allow room for the heads of the structural bolts under the Linear Track. Just as the LTBP, the Spacer is made from AL - 6061 for weight limitations (see Appendix D). The Sprocket (Figure 5), made of low carbon steel, has the same structural holes dilled through it, but it also has a grove machined out of the underside. This groove holds the press-fitted Thrust Bearing. The groove's clearance for the thrust bearing is about 0.002 in. and 1/8 in. into the surface of the Sprocket. Also holding the Thrust Bearing is the Thrust Bearing Column (TBC, Figure 6). The TBC supports the entire model through the Thrust Bearing and similar to the Sprocket has about 0.002 in. clearance for the Bearing. The TBC is bolted to the Triangular Baseplate, a part of the Project completed two years ago by Damon Bennett. The final design procedure of the assembly is to choose the bolts. Based upon the design criteria, Grade 8, heat treated (Appendix E) bolts with a

diameter of 5/8 in. are chosen due to their strength. Through the center of the assembly, in order to hold the emergency catcher,

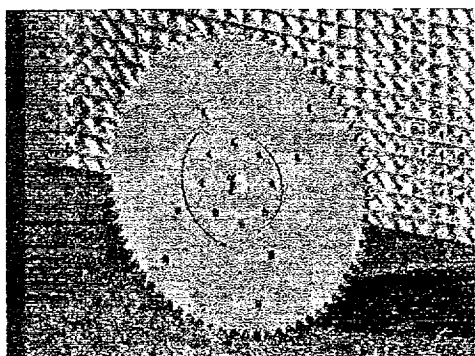


Figure 5 - Driving Sprocket

completely threaded 5/8 in. rod holds the Turntable together. These rods, although grade 8 steel, have the strength of grade 5 because of the thread. These bolts were purchased at The Rule Company.



PROJECT VALIDATION PROCEDURES

To ascertain the actual strength performance of the LTBP certain analysis were implemented. Weighing the LTBP and measuring the deflection can allow one to calculate the modules of elasticity. Since the LTBP is 11 ft long, a 3-ton overhead hydraulic jack (above picture) with a spring scale attached to its end lifts the beam up until only the extremities touch the flat surface. This same procedure also measures the deflection at the extremities of the Track. The angle of twist cannot be easily determined experimentally because of the large amount of twist the track is designed to withstand. However, by knowing the modulus of elasticity the twist can be "experimentally" calculated.

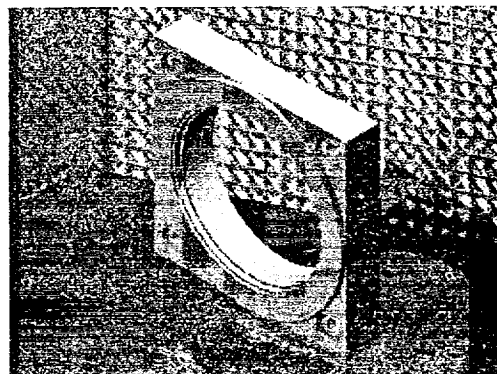


Figure 6 - Thrust Bearing Column

RESULTS

A manufactured final product has been the purpose of this project from its establishment. It is therefore only reasonable that the

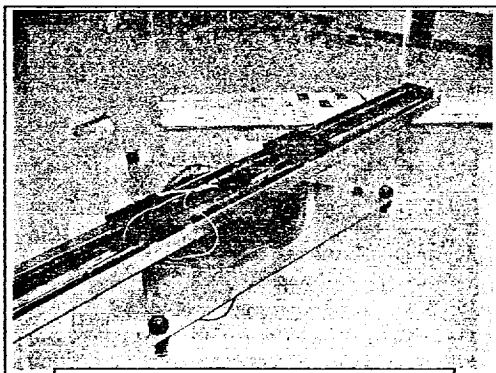


Figure 8 - Finished Assembly

structural support of the SRS be fabricated and constructed. Once the designs were developed (Appendix D), Waldon Machining Inc.² (WMI) produced the actual components. They bought the Aluminum "C" Channel, and together with material supplied by ORU, machined the Linear Track Baseplate, Driving Sprocket, Spacer and Thrust Bearing Column (figure 8). The design for the LTBP called for a 10*6.136, AL-6061 Channel, however at WMI's recommendation a Chamfered "C" AL-6061 Channel (see Appendix B, figure 9). This Channel has better twist and bending properties and corresponds to the original design specifications to a greater degree. Though the drafts were slightly incomplete, Waldon exceptionally produced the desired results and delivered them to ORU

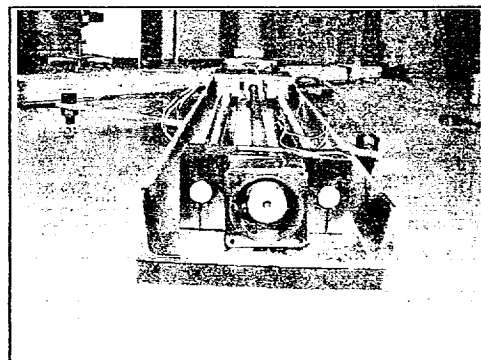


Figure 9 - Chamfered "C" Channel

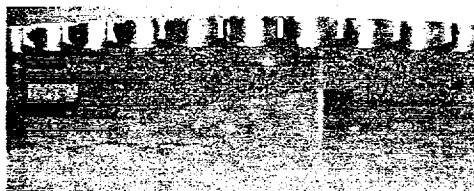


Figure 10 - Thrust Bearing Column

within three weeks. The Thrust Bearing Column's press fit accurately corresponded to the Thrust Bearing, however, the Sprocket needed to be re-machined to the correct specifications.

² Aerospace machining shop in Tulsa, OK

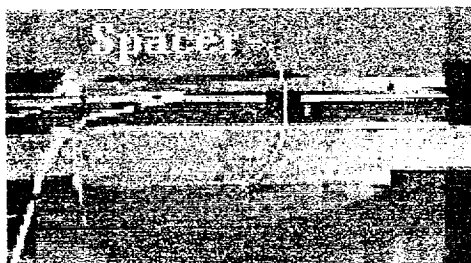


Figure 11 - Spacer As Is

This error was due to a misunderstanding of the assembly of the Turntable on the part of WMI. After being adjusted, the

Sprocket fit perfectly in the Thrust Bearing and spun uninhibited on top of the Thrust Bearing Column (Figure 10).

The Spacer, most complex of the machined parts (Figure 11), returned to ORU perfectly designed. Figure 11 is an actual photo of the Spacer mounted between the LTBP and Sprocket. In the completion of the Turntable System, other sub projects were completed. These subprojects include



Figure 12 - Components

the assembly of the Slip Rings, Airline for frictionless air bearing (Figure 12), Linear Track motor mount (Figure 9) and Catcher.

TABLE 1 - LTBP DESIGN SPECIFICATIONS

LTBP Design Specifications	Theoretical	Actual
Deflection	0.573 in. (1.46 cm)	
Weight	42.9 lb (19.5 kg)	
Angle of Twist	6.82 deg	

PROJECT MANAGEMENT

The progress of this project did not proceed according to the initial belief that design would be sequential. One of the greatest lessons learned in managing this project was that the design phase did not proceed one piece after another. Rather, all components derived their design from each other and therefore needed to be designed simultaneously. In the planning stages of this project, many hours were spent sitting in the SRS lab and breaking down all the tasks that needed to be accomplished from the floor to the Model. In order to grasp a better picture of what was really going on each task was outlined and scheduled with predicted times. The table below shows the planning that went into this phase.

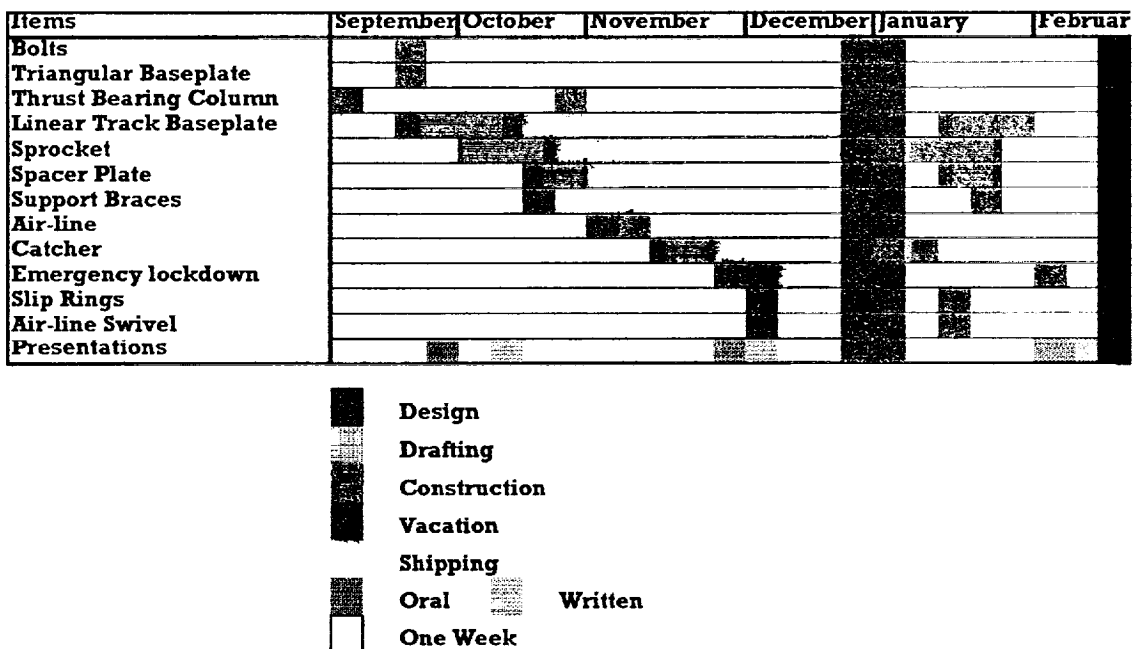
TABLE 2 - PLANNING PROCEDURE

ITEM	DESCRIPTION
BOLTS	De-rust by sanding and set nuts at correct height, purchase necessary bolts for design from Lowe's
TRIANGULAR BASEPLATE	Affix in place and level to secure the turntable at correct plane.
THRUST BEARING COLUMN	Go over and update designs. Machine to specifications. Check stress calculations
LINEAR TRACK BASEPLATE	Design, analyze the strength of material needed, purchase from supplier, outsource machining to Waldon's
SPROCKET	Design bolt patterns and drill using mill, press fit Thrust Bearing.
SPACER PLATE	Calculate height requirement of the Spacer to clear

	chain and recess bolt heads.
SUPPORT BRACES	Calculate necessary support braces to decrease angle of twist and deflection.
AIRLINE	Plumb air line for the Spherical Air-Bearing
CATCHER	Design safety device to protect all from the inopportune event that the Model would fly off the Support Column. Disk break under Triangular Baseplate
EMERGANCY LOCKDOWN	Design mechanism to quickly halt system if too much instability occurs. I.e. cut-off switch

All of this information was linearly placed into an Excel spreadsheet to organize it in time.

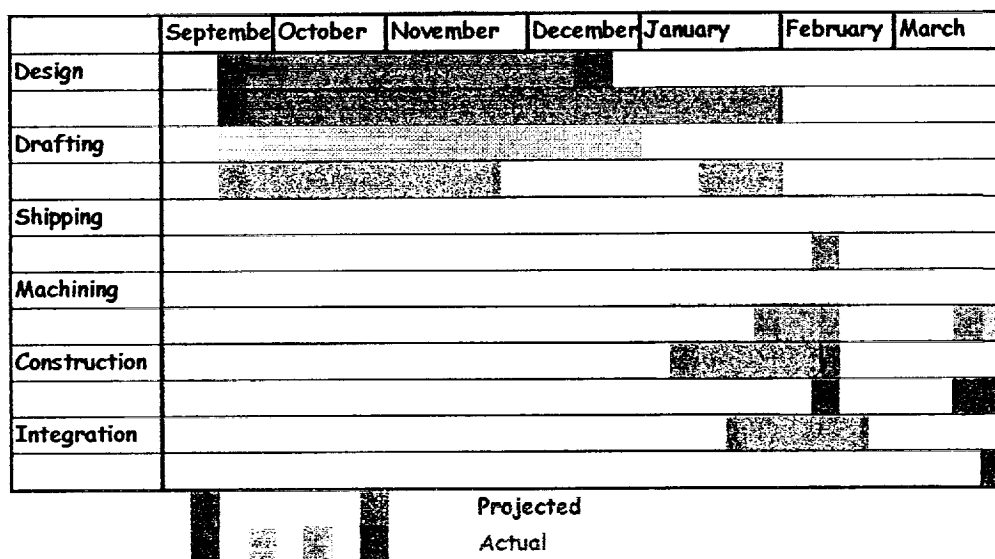
TABLE 3 - INITIAL SCHEDULE



The above schedule exhibits the errant sequential design and production of the Turntable System. As fore stated, the actual design

and analysis transpired concurrently which led to the following time schedule. Included in the darker hue is the actual time line of design, drafting, manufacturing, and assembling.

TABLE 4 - REVISED AND ACTUAL SCHEDULE



The budget progressed through similar modifications as the schedule, but unlike the schedule it ended up looking more like the original. The first budget was proposed in August to ORU and accepted as such.

TABLE 5 - PROJECTED BUDGET

Linear Track Base-Plate	\$500.00
Machining of Sprocket	\$500.00
Machining of Base Plate	\$500.00
Lab Instruments	\$100.00
Nuts and Bolts	\$50.00

Welding and Soldering	\$50.00
<u>Miscellaneous</u>	<u>\$100.00</u>
Total	\$1800.00

This budget was based upon researching previous projects of similar caliber. Some of the differences come in how the money was distributed. Instead of performing the minor machining at ORU, Waldon was offered a bulk price to machine all parts and purchase those that were needed. The parts produced by Waldon Machining include, the Linear Track Baseplate (they purchased the "C" Channel), the Thrust Bearing Column, the Driving Sprocket, the Spacer, and the finishing of other minor items. The Bolts were purchased at both Lowe's hardware store and The Rule Company³.

TABLE 6 - ACTUAL BUDGET

Aluminum "C" Channel	\$200.00
Machining of Spacer, Sprocket & "C" Channel	\$1800.00
Bolts	\$125.00
<u>Miscellaneous</u>	<u>\$100.00</u>
Total	\$2225.00

This budget accurately summarizes the expenditures of the Turntable Project. Learning the management of this project has been one of the greatest lessons obtained. Through planning tasks, analyzing procedure, formulating a budget and actually walking out this project, skills and administrative talents have developed.

³ Bolt specialist in Tulsa, OK

CONCLUSIONS AND RECOMMENDATIONS

The Turntable Design adequately supports the Satellite Model, however, more integration is required for a completely functional spinning rocket simulator. The LTBP counters torsion and bending caused by the maximum design forces and torques. It is

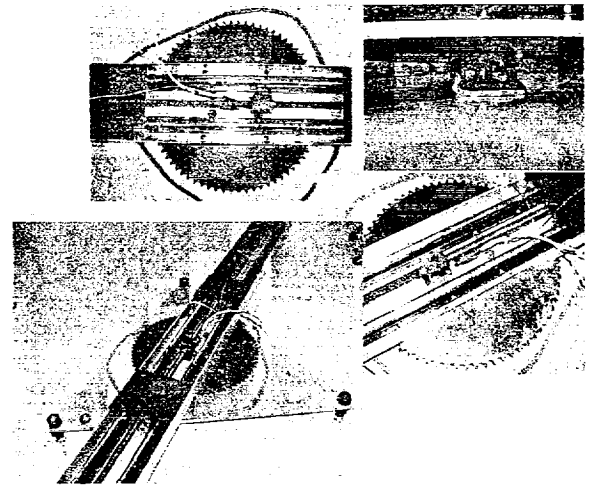


Figure 13 - Complete Design

recommended that the Catcher be completed and installed as well as adjusting the Airline.

Because the Linear Track's fastening holes are off center the entire track is not totally balanced. This will need to be taken into account when designing the Counter Balance. The wires and flexible air hose must be installed after the Model Support and Counter Balance have been mounted.

ONE LAST PICTURE

One last picture that I want to include is a picture of Elly Burns and myself. I also want to thank Jesus Christ for the endurance and perseverance in completing this paper having lost all my research 3 weeks ago.



REFERENCES

Those listed below have been of invaluable help and assistance.

Thompson Industries	-	Linear Track
Liz	-	(516) 944 - 1191
Waldon Machining Inc.	-	All Machining
Rick Thomas	-	(918) 836 - 6317
Solidworks, Cosmosworks	-	All drafting and computer design
Clay Slaton	-	(918) 599 - 7500
The Rule Company	-	All bolts and threaded fasteners
	-	(918) 585 - 5757

APPENDIX A

relation between the height of the Linear Track Base Plate and the angle of twist of that plate.

Height of Base Plate		Linear Track Angle of Twist		Horizontal Displacement		Spherical Air Bearing Vertical Displacement		Mass of Plate	
in.	m	Radians	Degrees	in.	cm	in.	cm	Kg	
0	0	0.55496	31.79684	18.614	47.27953	5.30178	13.46652	0	
.125	0.003175	0.30902	17.70552	10.782	27.38568	1.679273	4.265353	13.70882	30.
.25	0.00635	0.214244	12.27529	7.5639	19.2124	0.813378	2.065981	27.41764	60.
.375	0.009525	0.163954	9.393867	5.8273	14.80123	0.478774	1.216085	41.12646	90.
.5	0.0127	0.132734	7.605084	4.7415	12.04333	0.315139	0.800454	54.83528	120
.625	0.015875	0.111432	6.38458	3.9979	10.15464	0.222977	0.566362	68.5441	151
.75	0.01905	0.095945	5.497268	3.4561	8.778475	0.165925	0.421451	82.25293	181
.875	0.022225	0.084161	4.822042	3.0432	7.729636	0.128133	0.325457	95.96175	211
1	0.0254	0.074878	4.29017	2.7175	6.902499	0.101788	0.258542	109.6706	241
.125	0.028575	0.067365	3.859746	2.4537	6.232464	0.082679	0.210005	123.3794	272
.25	0.03175	0.061152	3.503771	2.2354	5.677816	0.06837	0.17366	137.0882	302
.375	0.034925	0.055922	3.204067	2.0513	5.210426	0.057372	0.145725	150.797	332
1.5	0.0381	0.051451	2.947948	1.894	4.81064	0.048734	0.123784	164.5059	362
.625	0.041275	0.047583	2.726295	1.7576	4.464319	0.041824	0.106232	178.2147	392
.75	0.04445	0.044198	2.532381	1.6382	4.161028	0.036209	0.09197	191.9235	423
.875	0.047625	0.04121	2.361141	1.5326	3.892903	0.031584	0.080224	205.6323	453
2	0.0508	0.038549	2.208684	1.4385	3.653907	0.027731	0.070436	219.3411	483
.125	0.053975	0.036163	2.071976	1.3541	3.439327	0.024486	0.062195	233.05	513
.25	0.05715	0.03401	1.948613	1.2777	3.245431	0.02173	0.055193	246.7588	54
.375	0.060325	0.032056	1.836668	1.2084	3.069228	0.019369	0.049198	260.4676	574
2.5	0.0635	0.030274	1.734577	1.145	2.908288	0.017333	0.044026	274.1764	604
.625	0.066675	0.028642	1.641057	1.0869	2.760624	0.015566	0.039537	287.8852	634
.75	0.06985	0.027141	1.555046	1.0333	2.624587	0.014023	0.035619	301.5941	664
.875	0.073025	0.025755	1.475657	0.9838	2.498802	0.012669	0.03218	315.3029	695
3	0.0762	0.024472	1.402144	0.9378	2.382113	0.011476	0.029149	329.0117	725
.125	0.079375	0.02328	1.333872	0.8951	2.273539	0.01042	0.026466	342.7205	755
3.25	0.08255	0.022171	1.270301	0.8552	2.172241	0.009481	0.024081	356.4293	785
.375	0.085725	0.021135	1.210966	0.8179	2.077501	0.008644	0.021955	370.1382	816
3.5	0.0889	0.020167	1.155462	0.783	1.988696	0.007895	0.020053	383.847	846
.625	0.092075	0.019259	1.10344	0.7501	1.905286	0.007223	0.018347	397.5558	876
3.75	0.09525	0.018406	1.054594	0.7192	1.826797	0.006619	0.016813	411.2646	906
.875	0.098425	0.017604	1.008653	0.6901	1.752815	0.006074	0.015429	424.9734	936
4	0.1016	0.016849	0.965379	0.6626	1.682971	0.005582	0.014179	438.6823	967

5.4 ANGLE OF TWIST

Occasionally the design of a shaft depends on restricting the amount of rotation or twist that may occur when the shaft is subjected to a torque. Furthermore, being able to compute the angle of twist for a shaft is important when analyzing the reactions on statically indeterminate shafts.

In this section we will develop a formula for determining the *angle of twist* ϕ (phi) of one end of a shaft with respect to its other end. The shaft is assumed to have a circular cross section that can gradually vary along its length, Fig. 5-15a, and the material is assumed to be homogeneous and to behave in a linear-elastic manner when the torque is applied. As in the case of an axially loaded bar, we will neglect the localized deformations that occur at points of application of the torques and where the cross section changes abruptly. By Saint-Venant's principle, these effects occur within small regions of the shaft's length and gradually have only a slight effect on the final result.

Using the method of sections, a differential disk of thickness dx , located at position x , is isolated from the shaft, Fig. 5-15b. The internal resultant torque is represented as $T(x)$, since the external loading may cause it to vary along the axis of the shaft. Due to $T(x)$ the disk will twist, such that the *relative rotation* of one of its faces with respect to the other face is $d\phi$, Fig. 5-15b. Furthermore, as explained in Sec. 5.1, an element of material located at an arbitrary radius ρ within the disk will undergo a shear strain γ . The values of γ and $d\phi$ are related by Eq. 5-1, namely,

$$d\phi = \gamma \frac{dx}{\rho} \quad (5-13)$$

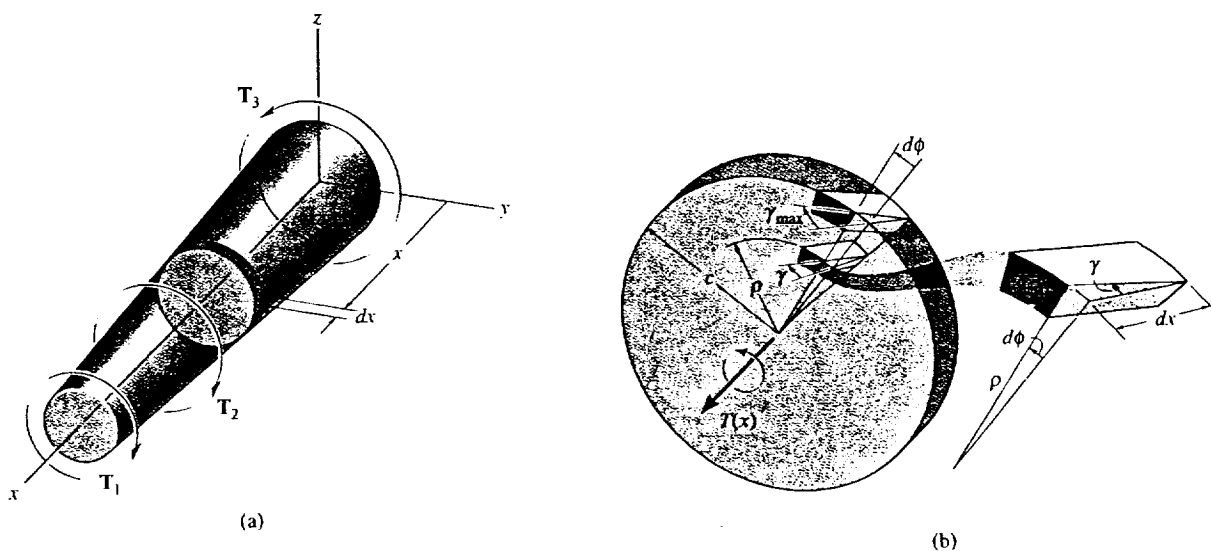


Fig. 5-15

Since Hooke's law, $\gamma = \tau/G$, applies, and the shear stress can be expressed in terms of the applied torque using the torsion formula $\tau = T(x)\rho/J(x)$, then $\gamma = T(x)\rho/J(x)G$. Substituting this result into Eq. 5-13, the angle of twist for the disk is

$$d\phi = \frac{T(x)}{J(x)G} dx$$

Integrating over the entire length L of the shaft, we obtain the angle of twist for the entire shaft, namely,

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G} \quad (5-14)$$

Here

ϕ = the angle of twist of one end of the shaft with respect to the other end, measured in radians

$T(x)$ = the internal torque at the arbitrary position x , found from the method of sections and the equation of moment-equilibrium applied about the shaft's axis

$J(x)$ = the shaft's polar moment of inertia expressed as a function of position x

G = the shear modulus for the material

Constant Torque and Cross-Sectional Area. Usually in engineering practice the material is homogeneous so that G is constant. Also, the shaft's cross-sectional area and the applied torque are constant along the length of the shaft, Fig. 5-16. If this is the case, the internal torque $T(x) = T$, the polar moment of inertia $J(x) = J$, and Eq. 5-14 can be integrated, which gives

$$\phi = \frac{TL}{JG} \quad (5-15)$$

The similarities between the above two equations and those for an axially loaded bar ($\Delta = \int P(x) dx/A(x)E$ and $\Delta = PL/AE$) should be noted.

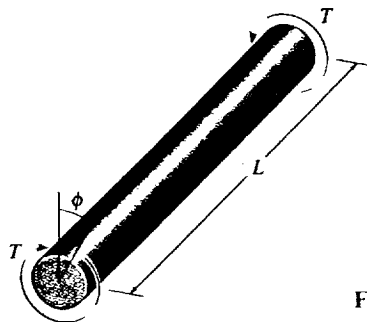


Fig. 5-16

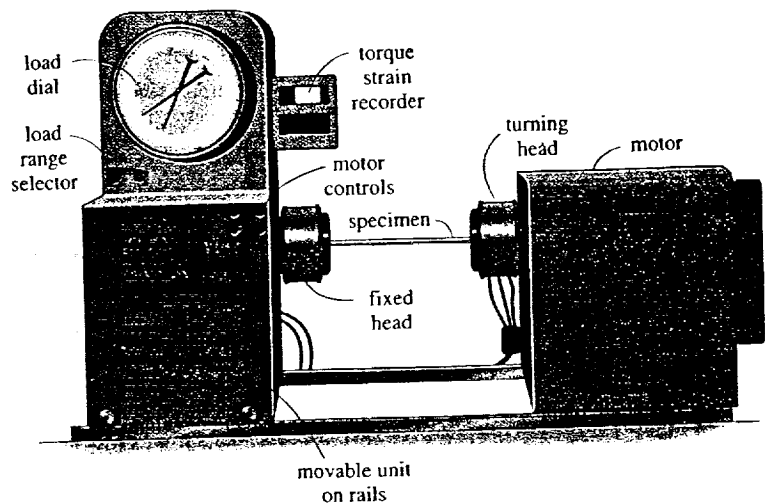
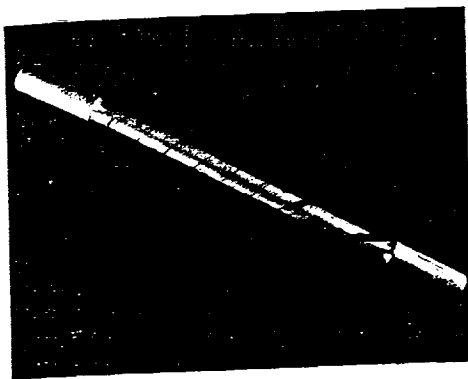


Fig. 5-17



Severe twist of an aluminum specimen

In particular, Eq. 5-15 is often used to determine the shear modulus G . To do so a specimen of known length and diameter is placed in a torsion testing machine like the one shown in Fig. 5-17. The torque T and angle of twist ϕ are then measured between a gauge length L . Using Eq. 5-15, $G = TL/J\phi$. Usually, to obtain a more reliable value of G , several of these tests are performed and the average value is used.

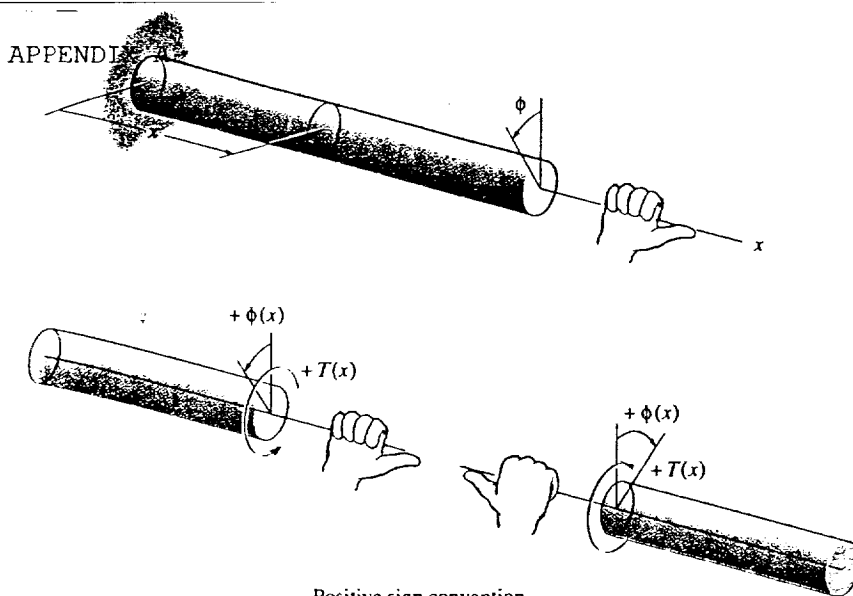
If the shaft is subjected to several different torques, or the cross-sectional area or shear modulus changes abruptly from one region of the shaft to the next, Eq. 5-15 can be applied to each segment of the shaft where these quantities are all constant. The angle of twist of one end of the shaft with respect to the other is then found from the vector addition of the angles of twist of each segment. For this case,

$$\phi = \sum \frac{TL}{JG}$$

(5-16)

Sign Convention. In order to apply the above equations, we must develop a sign convention for the internal torque and the angle of twist of one end of the shaft with respect to the other end. To do this we will use the right-hand rule, whereby both the torque and angle will be *positive* provided the *thumb* is directed *outward* from the shaft when the fingers curl to give the tendency for rotation, Fig. 5-18.

To illustrate the use of this sign convention, consider the shaft shown in Fig. 5-19a, which is subjected to four torques. The angle of twist of end A with respect to end D is to be determined. For this problem, three segments of the shaft must be considered, since the internal torque changes at B and C . Using the method of sections, the internal



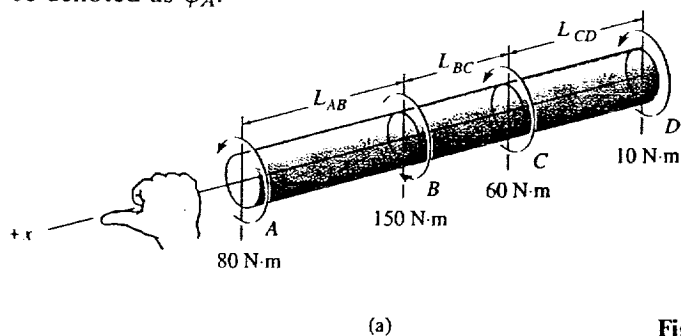
Positive sign convention
for T and ϕ

Fig. 5-18

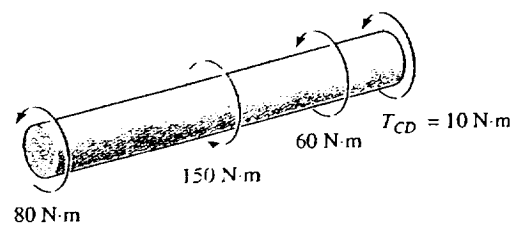
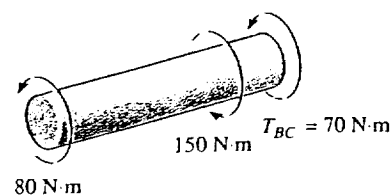
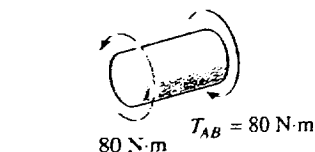
torques are found for each segment, Fig. 5-19b. By the right-hand rule, with positive torques directed away from the *sectioned end* of the shaft, we have $T_{AB} = +80 \text{ N} \cdot \text{m}$, $T_{BC} = -70 \text{ N} \cdot \text{m}$, and $T_{CD} = -10 \text{ N} \cdot \text{m}$. These results are also shown on the *torque diagram* for the shaft, Fig. 5-19c. Applying Eq. 5-16, we have

$$\phi_{A/D} = \frac{(+80 \text{ N} \cdot \text{m}) L_{AB}}{JG} + \frac{(-70 \text{ N} \cdot \text{m}) L_{BC}}{JG} + \frac{(-10 \text{ N} \cdot \text{m}) L_{CD}}{JG}$$

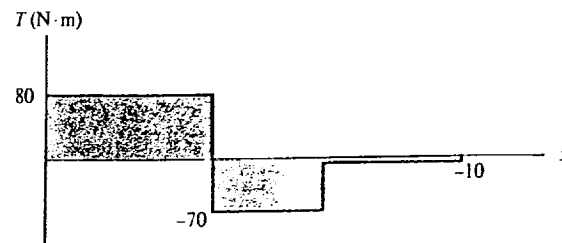
If the other data is substituted and the answer is found as a *positive* quantity, it means that end A will rotate as indicated by the curl of the right-hand fingers when the thumb is directed *away* from the shaft, Fig. 5-19a. The double subscript notation is used to indicate this relative angle of twist ($\phi_{A/D}$); however, if the angle of twist is to be determined relative to a *fixed point*, then only a single subscript will be used. For example, if D is located at a fixed support, then the computed angle of twist will be denoted as ϕ_A .



(a)



(b)



(c)

Fig. 5-19

5.5 STATICALLY INDETERMINATE TORQUE-LOADED MEMBERS

A torsionally loaded shaft may be classified as statically indeterminate if the moment equation of equilibrium, applied about the axis of the shaft, is not adequate to determine the unknown torques acting on the shaft. An example of this situation is shown in Fig. 5-24. As shown on the free-body diagram, the reactive torques at the supports A and B are unknown. We require

$$\Sigma M_x = 0; \quad T - T_A - T_B = 0$$

Since only one equilibrium equation is relevant and there are two unknowns, this problem is statically indeterminate. In order to obtain a solution we will use the method of analysis discussed in Sec. 4.4.

The necessary condition of compatibility, or the kinematic condition, requires the angle of twist of one end of the shaft with respect to the other end to be equal to zero, since the end supports are fixed. Therefore,

$$\phi_{A/B} = 0$$

In order to write this equation in terms of the unknown torques, we will assume that the material behaves in a linear-elastic manner, so that the load-displacement relationship is expressed by $\phi = TL/JG$. Realizing that the internal torque in segment AC is $+T_A$, and in segment CB the internal torque is $-T_B$, the above compatibility equation can be written as

$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0$$

Here JG is assumed to be constant.

Solving the above two equations for the reactions, realizing that $L = L_{AC} + L_{BC}$, we get

$$T_A = T \left(\frac{L_{BC}}{L} \right)$$

and

$$T_B = T \left(\frac{L_{AC}}{L} \right)$$

Note that each reactive torque increases or decreases linearly with the distance T is from each support.

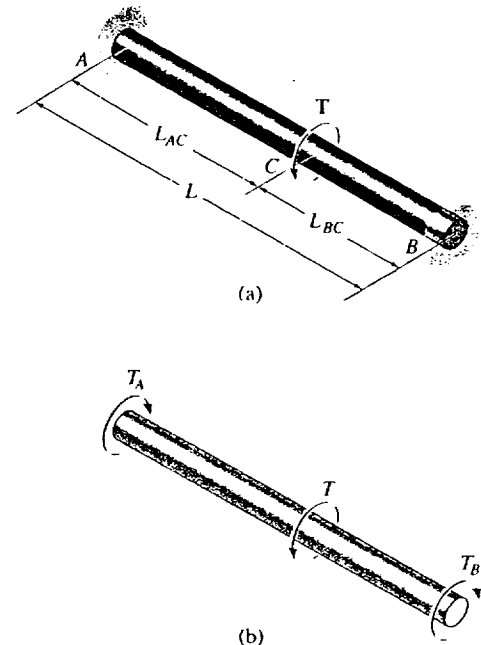
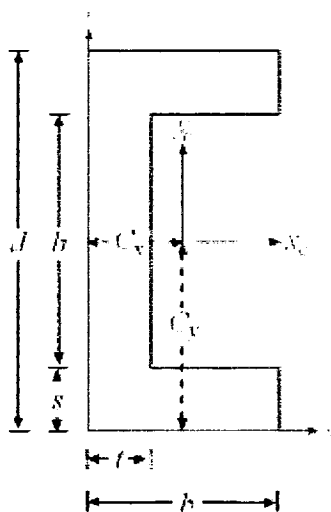


Fig. 5-24

APPENDIX B

AMERICAN STANDARD "C" CHANNEL



$$C_x = \frac{2b^2s + ht^2}{2bd - h(b-t)}$$

$$C_y = \frac{d}{2}$$

Area

$$bd - h(b-t)$$

Moment of Inertia
about the x_c axis
 I_{xc}

Moment of Inertia
about the y_c axis
 I_{yc}

Polar Moment of Inertia
about the z_c axis
 J_{zc}

$$\frac{bd^3 - h^3(b-t)}{12}$$

$$\frac{2sb^3 + ht^3}{3} - AC_x^2$$

where $A = 2sb + ht$

$$= I_{xc} + I_{yc}$$

Radius of Gyration
about the x_c axis $\langle BR \rangle k_{xc}$

Radius of Gyration
about the y_c axis
 k_{yc}

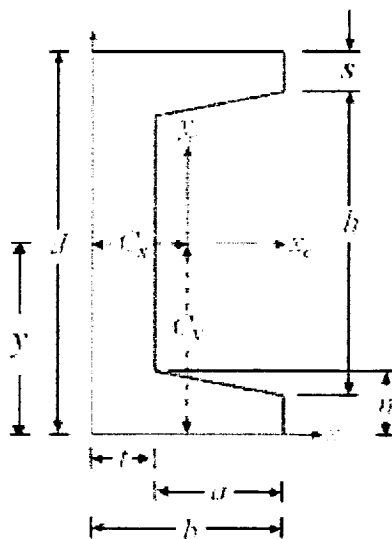
Radius of Gyration
about the z_c axis
 r_{zc}

$$\sqrt{\frac{bd^3 - h^3(b-t)}{12[bd - h(b-t)]}}$$

$$\sqrt{\frac{I_{yc}}{A}}$$

$$r_{zc}^2 = k_{xc}^2 + k_{yc}^2$$

CHAMFERED "C" CHANNEL



C_x	C_y	Area
$\frac{1}{A} \left[b^2 s + \frac{ht^2}{2} + \frac{g}{3} (b+2t)(b-t)^2 \right]$	$\frac{d}{2}$	$dt + a(s+n)$
$g = \frac{h-L}{2(b-t)}$		
<hr/>		
Moment of Inertia about the x_c axis I_{xc}	Moment of Inertia about the y_c axis I_{yc}	Polar Moment of Inertia about the z_c axis J_{zc}
$\frac{1}{12} \left[bd^3 - \frac{1}{8g} (h^4 - L^4) \right]$	$\frac{1}{3} \left[2sb^3 + Lt^3 + \frac{g}{2} (b^4 - t^4) \right] - A(b-y)^2$	
$g = \text{slope of Flange} = \frac{h-L}{2(b-t)}$	$g = \frac{h-L}{2(b-t)}$	$= I_{xc} + I_{yc}$
For Standard Channels $g = \frac{1}{6}$		

APPENDIX B

Radius of Gyration
about the x_c axis
 k_{xc}

$$\sqrt{\frac{\frac{1}{12} \left[bd^3 - \frac{1}{8g} (h^4 - L^4) \right]}{dt + a(s+n)}}$$

Radius of Gyration
about the y_c axis
 k_{yc}

$$\sqrt{\frac{\frac{1}{3} \left[2sb^3 + Lt^3 + \frac{g}{2} (b^4 - t^4) \right] - A(b-y)^2}{dt + a(s+n)}}$$

Radius of Gyration
about the z_c axis
 r_{zc}

$$r_{zc}^2 = k_{xc}^2 + k_{yc}^2$$

APPENDIX C

>
Linear Track Base Plate
C10 x 15.3 Channel
Angle of twist.

> restart;

Equations needed for a Steel C10 x 15.3 Channel

> d := 10 * .0254:
> A := 4.49 * .0254^2:
> bf := 2.6 * .0254:
> tf := .436 * .0254:
> tw := .24 * .0254:
> Cx := .634 * .0254:
> Ixx := 67.4 * .0254^4:
> Iyy := 2.28 * .0254^4:
> Jb := Ixx + Iyy;

$$Jb := .00002900300574$$

> v := 0.3:
> Eb := 207e9:
> Gb := Et / (2 * (1 + v));

$$Gb := .7961538462 \cdot 10^{11}$$

The next section has to do with the linear track itself.

> Ea := 12.817e9:
> Ja := 3.0153e-5:
> Ga := Ea / (2 * (1 + v));

$$Ga := .4929615385 \cdot 10^{10}$$

The following equations deal with the angle of twist and the system in general.

> L := 1.433:
> Hcol := .810:
> y := ((.058 + tw) - (((.178 * .058) * .058 / 2) + A * Cx) / ((.178 * .058) + A))
+ Hcol;

$$y := .8479216969$$

> To := 67434 * y;

APPENDIX C

$$T_o := 57056.60297$$

> Ta := To - Tb:

> Ta := To - Tb;

$$T_a := 57056.60297 - T_b$$

> Tb := .simplify((Theta*(Jb*Gb))/L);

$$T_b := 690287.4999 \Theta$$

> Eqn2 := (Ta*L)/(Ja*Ga) - (Tb*L)/(Jb*Gb)=0;

$$Eqn2 := .5500580659 - 7.654763644 \Theta = 0$$

> Ang_rad := solve(Eqn2,Theta);

$$Ang_rad := .07185826911$$

> Ang_deg := simplify(Ang_rad*(180/Pi));

$$Ang_deg := 4.117175542$$

This is a check for the bending of the base plate assuming that the elasticity of the track is negligible.

> Wo := 207.3 + 223.287;

$$W_o := 430.587$$

> P := 552:

> Yt := -(Wo*L^4)/(8*Eb*Iyy) - (P*L^3)/(3*Eb*Iyy);

$$Y_t := -.004226742953$$

>

APPENDIX C

>
Linear Track Base Plate
Actual Aluminium "C" Channel
Angle of twist.

> restart;

This next set of equations and dimentions are associated with the channel shaped base plate.

> b := 2.6*.0254:
> t := .24*.0254:
> s := .24*.0254:
> d := 10*.0254:
> h := d - 2*s:
> A := b*d - h*(b-t):
> Cx := (2*b^2*s + h*t^2)/(2*b*d - 2*h*(b-t)):
> Ixx := (b*d^3 - (h^3)*(b-t))/12:
> Iyy := ((2*s*b^3 + h*t^3)/3) - A*Cx^2:
> Jb := Ixx +Iyy:

> v:=0.3:
> Eb := 69e9:
> Gb := Eb/(2*(1+v)):

The next section has to do with the linear track itself.

> Ea := 12.817e9:
> Ja := 3.0153e-5:
> Ga :=Ea/(2*(1+v)):

The following equations deal with the angle of twist and the system in general.

> L := 1.433:
> Hcol :=.810:
> y := ((.058+t) - (((.178*.058)*.058/2) + A*Cx)/((.178*.058)+A))
+Hcol:
> To := 67434*y;

To := 57175.56932
> Ta := To - Tb:
> Tb := simplify((Theta*(Jb*Gb))/L):
> Eqn2 := (Ta*L)/(Ja*Ga) - (Tb*L)/(Jb*Gb)=0:

APPENDIX C

```
> Ang_rad := solve(Eqn2, Theta):
> Ang_deg := simplify(Ang_rad*(180/Pi));
Ang_deg := 6.824056633
```

This is a check for the bending of the base plate assuming that the elasticity of the track is negligible.

```
> Wo := 207.3 + 223.287:
> P := 552:
> Yt := -(Wo*L^4)/(8*Eb*Iyy) - (P*L^3)/(3*Eb*Iyy);
Yt := -.01455788199
> Deflection_angle := tan(Yt/L);
Deflection_angle := -.01015937392
>
```


APPENDIX C

>

Linear Track Base Plate

10 x 6.136 6061 Aluminum Channel

Angle of twist.

> restart;

Equations needed for a 10 x 6.136 6061 Aluminum Channel

> d := 10 * .0254:

> A := 5.218 * .0254^2:

> bf := 3.5 * .0254:

> tf := .41 * .0254:

> tw := .25 * .0254:

> Cx := 1.02 * .0254:

> Ixx := 83.22 * .0254^4:

> Iyy := 6.33 * .0254^4:

> Jb := Ixx + Iyy;

Jb := .00003727352416

> v := 0.3:

> Eb := 69e9:

> Gb := Eb / (2 * (1 + v));

Gb := .2653846154 10¹¹

The next section has to do with the linear track itself.

> Ea := 12.817e9:

> Ja := 3.0153e-5:

> Ga := Ea / (2 * (1 + v));

Ga := .4929615385 10¹⁰

The following equations deal with the angle of twist and the system in general.

> L := 1.433:

> Hcol := .810:

> y := ((.058 + tw) - (((.178 * .058) * .058 / 2) + A * Cx) / ((.178 * .058) + A))
+ Hcol;

y := .8461103148

> To := 67434 * y;

APPENDIX C

```

                                To := 57056.60297
> Ta := To - Tb:
> Ta := To - Tb;
                                Ta := 57056.60297 - Tb
> Tb := simplify((Theta*(Jb*Gb))/L);
                                Tb := 690287.4999 @
> Eqn2 := (Ta*L)/(Ja*Ga) - (Tb*L)/(Jb*Gb) = 0;
                                Eqn2 := .5500580659 - 7.654763644 @ = 0
> Ang_rad := solve(Eqn2, Theta);
                                Ang_rad := .07185826911
> Ang_deg := simplify(Ang_rad*(180/Pi));
                                Ang_deg := 4.117175542

```

This is a check for the bending of the base plate assuming that the elasticity of the track is negligible.

```

> Wo := 207.3 + 223.287;
                                Wo := 430.587
> P := 552:
> Yt := -(Wo*L^4)/(8*Eb*Iyy) - (P*L^3)/(3*Eb*Iyy);
                                Yt := -.004226742953
>

```

APPENDIX C

>
Linear Track Base Plate
Actual Aluminium "C" Channel
Angle of twist.

> restart;

This next set of equations and dimentions are associated with the channel shaped base plate.

```
> b := 2.6*.0254:
> t := .24*.0254:
> s := .24*.0254:
> d := 10*.0254:
> h := d - 2*s:
> A := b*d - h*(b-t):
> Cx := (2*b^2*s + h*t^2)/(2*b*d - 2*h*(b-t)):
> Ixx := (b*d^3 - (h^3)*(b-t))/12:
> Iyy := ((2*s*b^3 + h*t^3)/3) - A*Cx^2:
> Jb := Ixx +Iyy:

> v:=0.3:
> Eb := 69e9:
> Gb := Eb/(2*(1+v)):
```

The next section has to do with the linear track itself.

```
> Ea := 12.817e9:
> Ja := 3.0153e-5:
> Ga :=Ea/(2*(1+v)):
```

The following equations deal with the angle of twist and the system in general.

```
> L := 1.433:
> Hcol :=.810:
> y := ((.058+t) - (((.178*.058)*.058/2) + A*Cx)/((.178*.058)+A))
+Hcol:
> To := 67434*y;

To := 57175.56932

> Ta := To - Tb:
> Tb := simplify((Theta*(Jb*Gb))/L):
> Eqn2 := (Ta*L)/(Ja*Ga) - (Tb*L)/(Jb*Gb)=0:
```

APPENDIX C

```
> Ang_rad := solve(Eqn2, Theta):
> Ang_deg := simplify(Ang_rad*(180/Pi));
Ang_deg := 6.824056633
```

This is a check for the bending of the base plate assuming that the elasticity of the track is negligible.

```
> Wo := 207.3 + 223.287:
> P := 552:
> Yt := -(Wo*L^4)/(8*Eb*Iyy) - (P*L^3)/(3*Eb*Iyy);
Yt := -.01455788199
> Deflection_angle := tan(Yt/L);
Deflection_angle := -.01015937392
>
```

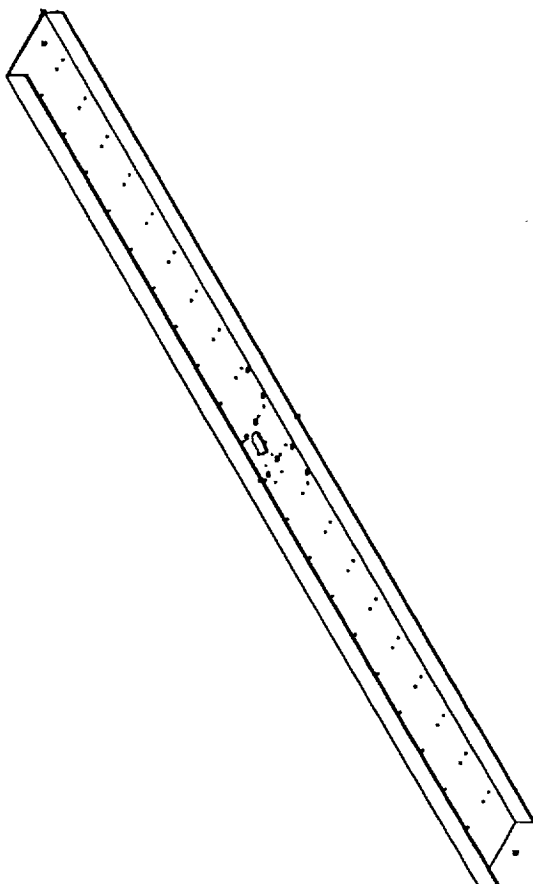
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(6.000)

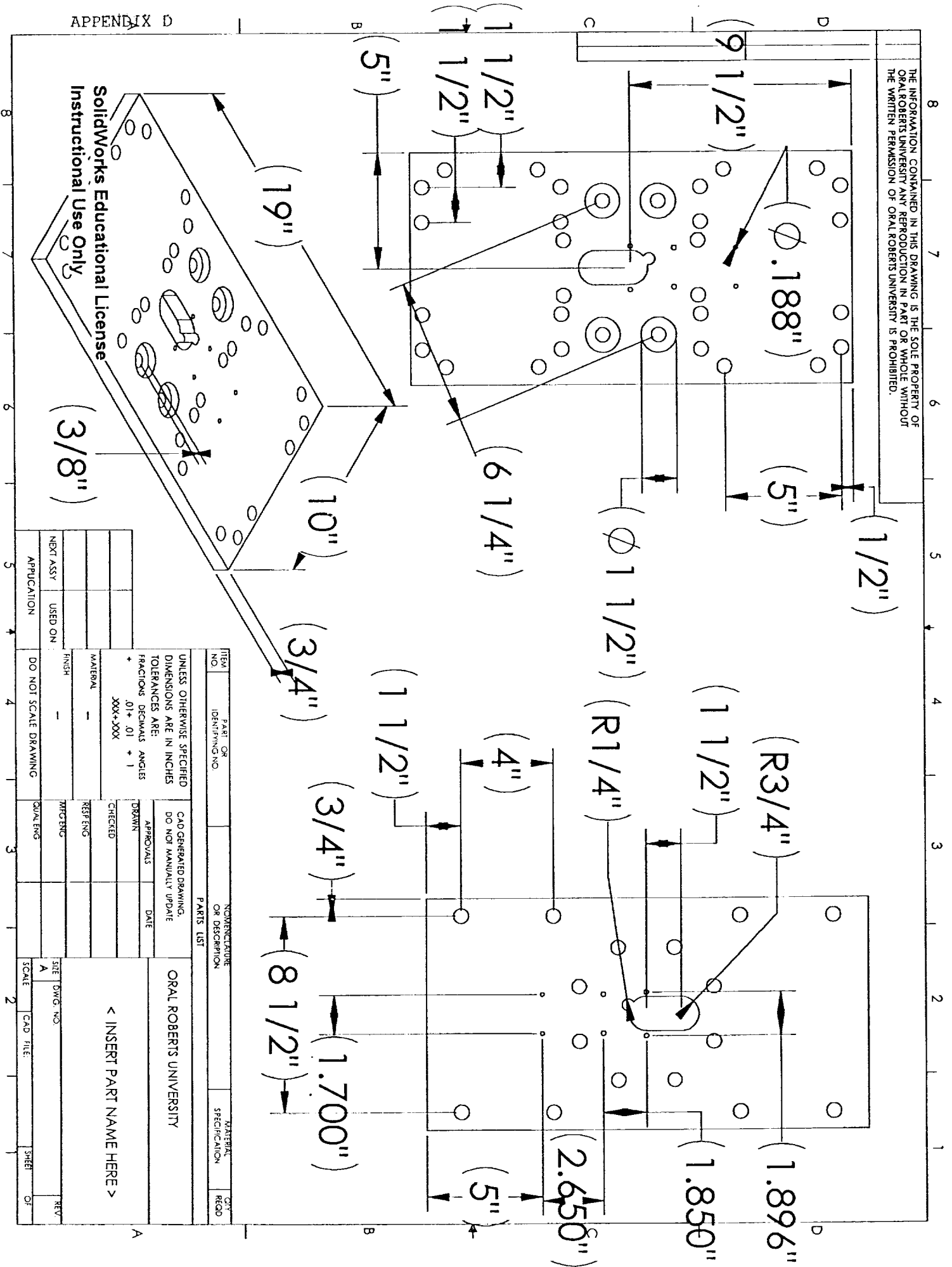
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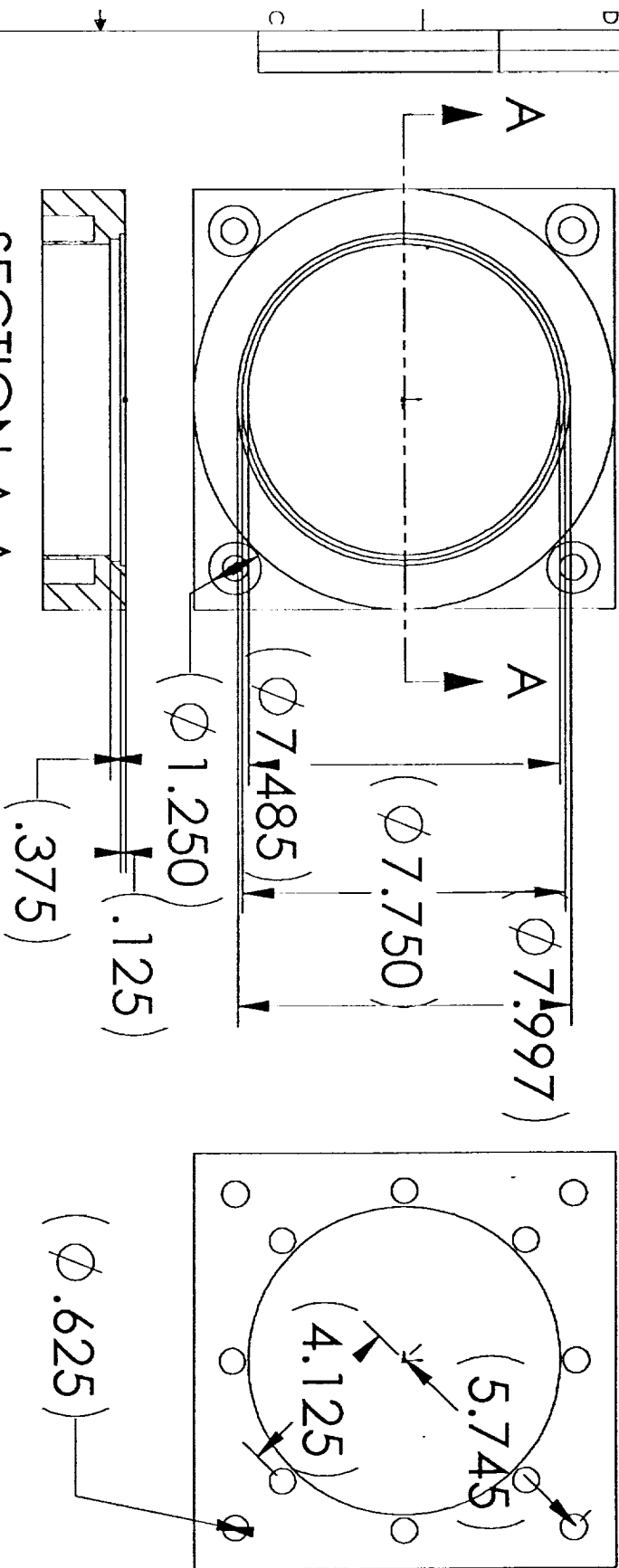
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PARTS LIST

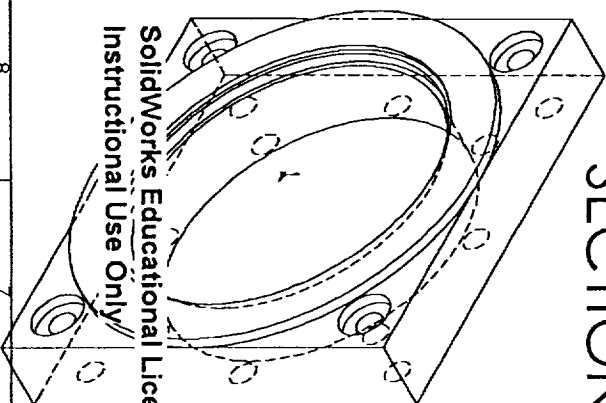
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SECTION A-A



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FINISH		WING ENG		
NEXT ASSY USED ON		SCALE ENG		
APPLICATION		DO NOT SCALE DRAWING		
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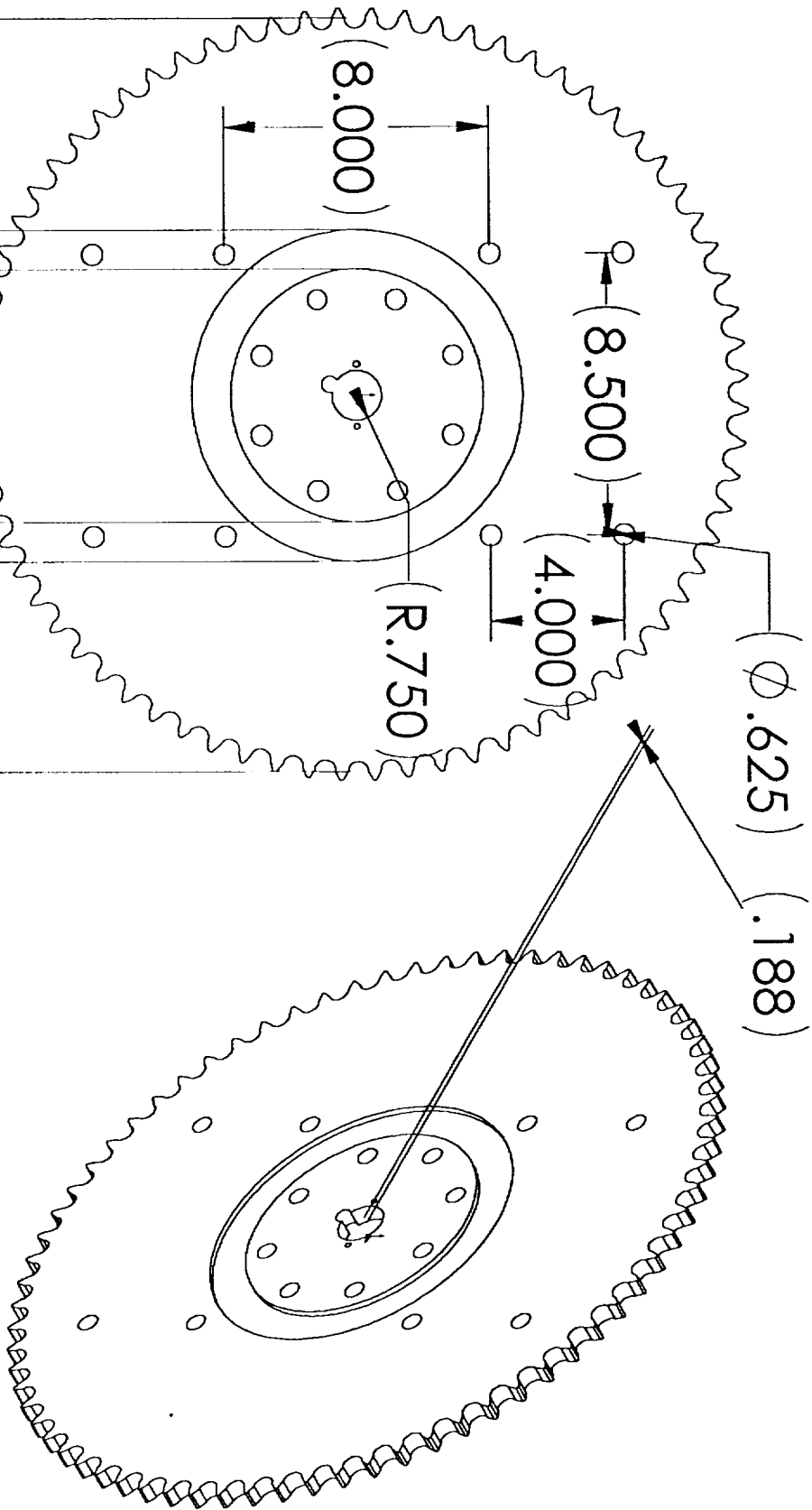
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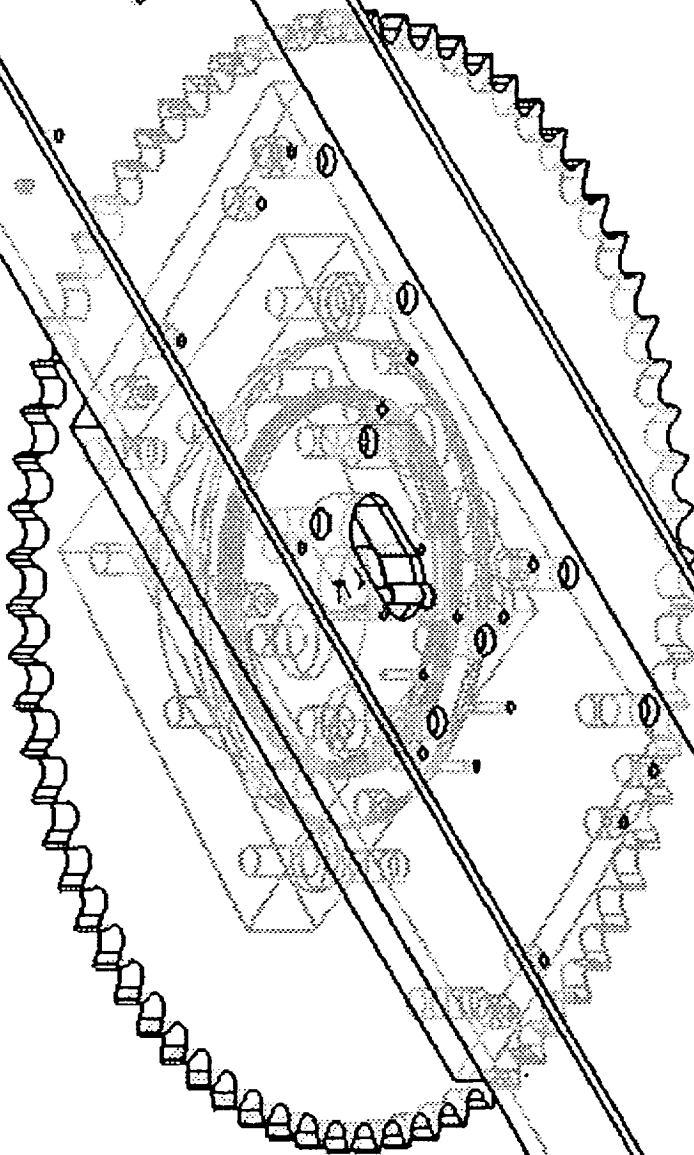


APPENDIX D

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APPLICATION		QUAL ENG	SCALE	
NEXT ASSY USED ON		REV.		

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APPENDIX E

Bolt Grade/Tensile Strength

C=coarse thread F=fine thread

Grade	Dia(in)	Coarse(lbs)	Fine(lbs)
2	1/4	2350	2700
	5/16	3900	4300
	7/16	7850	8800
	1/2	10500	11800
	9/16	13500	15000
	5/8	16700	18900
5	3/4	24700	27600
	1/4	3800	4350
	5/16	6300	6950
	7/16	12800	14400
	1/2	17000	19200
	9/16	21800	24400
	5/8	27100	30700

	3/4	40100	44800
8	1/4	4750	5450
	5/16	7850	8700
	7/16	15900	17800
	1/2	21300	24000
	9/16	27300	30400
	5/8	33900	38400
	3/4	50100	56000

C=coarse thread F=fine thread